

Entropy Production Due to Heat Flow

Consider two bodies or two phases maintained at temperatures T_1 and T_2 , with

$$T_2 > T_1$$

Let an infinitesimal quantity of heat $d_i q$ be transferred from phase II at T_2 to phase I at T_1 . Since heat is transferred from the higher temperature to the lower temperature, this is an irreversible process.

For a system undergoing an irreversible process, the entropy change is written as

$$dS = d_e S + d_i S$$

For heat transfer, the entropy exchanged with the surroundings is associated with the heat transfer term. For the two phases, the entropy changes resulting from the transfer of $d_i q$ are considered separately.

The entropy gained by phase I at temperature T_1 is

$$dS_1 = \frac{d_i q}{T_1}$$

Phase II loses the same amount of heat. Therefore, its entropy change is

$$dS_2 = -\frac{d_i q}{T_2}$$

The total entropy change resulting from the heat transfer is therefore

$$d_i S = dS_1 + dS_2$$

Substituting the two entropy changes,

$$d_i S = \frac{d_i q}{T_1} - \frac{d_i q}{T_2}$$

Taking $d_i q$ common,

$$d_i S = d_i q \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

Since $T_2 > T_1$,

$$\frac{1}{T_1} > \frac{1}{T_2}$$

and hence

$$d_i S > 0$$

This is consistent with the second law of thermodynamics: entropy production during an irreversible process is positive.

Rate of Entropy Production

Dividing the above equation by dt ,

$$\frac{d_i S}{dt} = \frac{d_i q}{dt} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

Therefore, the rate of entropy production is

$$\sigma = \frac{d_i q}{dt} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

where σ denotes the rate of entropy production.

This expression can be written in the general form

$$\sigma = J_q X_q$$

where

$$J_q = \frac{d_i q}{dt}$$

is the heat-flow flux and

$$X_q = \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

is the corresponding thermodynamic driving force.

Thus, the rate of entropy production is the product of a **flux** and its corresponding **driving force**.

Condition for Zero Entropy Production

From

$$d_i S = d_i q \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

entropy production becomes zero when

$$\frac{1}{T_1} - \frac{1}{T_2} = 0$$

Therefore,

$$\frac{1}{T_1} = \frac{1}{T_2}$$

and hence

$$\boxed{T_1 = T_2}$$

Thus, entropy production due to heat flow becomes zero only when the two phases reach **thermal equilibrium**.

At thermal equilibrium, there is no temperature difference and consequently no net heat flow caused by that temperature difference.

The result can therefore be summarized by

Temperature difference $\rightarrow$ heat flow $\rightarrow$ entropy production

and at equilibrium,

$T_1 = T_2 \quad \Rightarrow \quad J_q = 0 \quad \Rightarrow \quad d_i S = 0$
